Boolean Algebra

Exercise 2 (C)

1.

x_1	x_2	x_3	$x_1 \vee x_2$	$x_2 \vee x_3$	$f_1 = (x_1 \vee x_2) \vee x_3$	$f_2 = (x_1) \vee (x_1 \vee x_3)$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	0	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

from Column VI & VII it is clear that $f_1 \cong f_2$

2. (a) $f_1(x_1, x_2) = x_1 \vee x_2$ (b) $f_2(x_1, x_2) = x_1 \wedge x_2$ (c) $f_3(x_1) = x_1'$

(a)

x_1	x_2	$f_1 = (x_1 \vee x_2)$
0	0	0
0	1	1
1	0	1
1	1	1

(b)

x_1	x_2	$f_2 = (x_1 \wedge x_2)$
0	0	0
0	1	0
1	0	0
1	1	1

(c)

x_1	x_1'
0	1
0	1
1	0
1	0

3. $f(x_1, x_2, x_3) = (x_1 \vee x_2) \wedge (x_1' \vee x_2') \wedge (x_2 \vee x_3)'$

x_1	x_2	x_3	x_1'	x_2'	$x_1 \vee x_2$	$x_1' \vee x_2'$	$x_2 \vee x_3$	$(x_2 \vee x_3)'$	$f(x_1, x_2, x_3)$
0	0	0	1	1	0	1	0	1	0
0	0	1	1	1	0	1	1	0	0
0	1	0	1	0	1	1	1	0	0
0	1	1	1	0	1	1	1	0	0
1	0	0	0	1	1	1	0	1	1
1	0	1	0	1	1	1	1	0	0
1	1	0	0	0	1	0	1	0	0
1	1	1	0	0	1	0	1	0	0

4. Ordered table of three ~~ordered~~ variables is

x_1	x_2	x_3	x_1'	x_2'	x_3'	$f_1 = x_1 \wedge x_2'$		$f_2 = x_2 \vee x_3'$		$(x_1 \vee x_2)'$	$x_1' \wedge x_3$	$f_3 = (x_1 \vee x_2)' \vee (x_1' \wedge x_3)$	min term
0	0	0	1	1	1	0	m_0	1	m_0	1	0	1	m_0
0	0	1	1	1	0	0	m_1	0	m_1	1	1	1	m_1
0	1	0	1	0	1	0	m_2	1	m_2	0	0	0	m_2
0	1	1	1	0	0	0	m_3	1	m_3	0	1	1	m_3
1	0	0	0	1	1	1	m_4	1	m_4	0	0	0	m_4
1	0	1	0	1	0	1	m_5	0	m_5	0	0	0	m_5
1	1	0	0	0	1	0	m_6	1	m_6	0	0	0	m_6
1	1	1	0	0	0	0	m_7	1	m_7	0	0	0	m_7

a) Sum of product (SOP) cannonical form in three variable x_1, x_2, x_3 of $f_1(x_1 x_2 x_3) = x_1 \wedge x_2'$ is

$m_4 \vee m_5 = (x_1 \wedge x_2' \wedge x_3') \vee (x_1 \wedge x_2' \wedge x_3)$

(b) SOP Cannonical form of $f_2(x_1 x_2 x_3) = x_2 \vee x_3'$ is

$m_0 \vee m_2 \vee m_3 \vee m_4 \vee m_6 \vee m_7$

ie $(x_1' \wedge x_2' \wedge x_3') \vee (x_1' \wedge x_2 \wedge x_3') \vee (x_1' \wedge x_2 \wedge x_3) \vee (x_1 \wedge x_2' \wedge x_3') \vee$
$(x_1 \wedge x_2 \wedge x_3') \vee (x_1 \wedge x_2 \wedge x_3)$

(C) SOP Cannionical form or DNP of $f_3 = (x_1 \vee x_2)' \vee (x_1' \wedge x_3)$ is

$m_0 \vee m_1 \vee m_3 = (x_1' \wedge x_2' \wedge x_3') \vee (x_1' \wedge x_2' \wedge x_3) \vee (x_1' \wedge x_2 \wedge x_3')$

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5) Ordered table of three variables for the following functions is:

x_1	x_2	x_3	$x_2 \vee x_3$	MAX TERMS	$x_2 \wedge x_3$	MAX TERMS
0	0	0	0	M_0	0	M_0'
0	0	1	1	M_1	0	M_1'
0	1	0	1	M_2	0	M_2'
0	1	1	1	M_3	1	M_3'
1	0	0	0	M_4	0	M_4'
1	0	1	1	M_5	0	M_5'
1	1	0	1	M_6	0	M_6'
1	1	1	1	M_7	1	M_7'

(a) Product of Sum (POS) Cannonical form or CN form of $x_2 \vee x_3$

$$M_0 \wedge M_4 = (x_1 \vee x_2 \vee x_3) \wedge (x_1' \vee x_2 \vee x_3)$$

(b) POS or CN form of $x_2 \wedge x_3$ is

$$M_0' \wedge M_1' \wedge M_2' \wedge M_4' \wedge M_5' \wedge M_6$$

$$= (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee x_3') \wedge (x_1 \vee x_2' \vee x_3) \wedge (x_1' \vee x_2 \vee x_3)$$
$$\wedge (x_1' \vee x_2 \vee x_3') \wedge (x_1' \vee x_2' \vee x_3)$$

[For POS form take the meet of all maxterms corresponding to 0 value of the function as in a Maxterm if value of Variab is '1' then it is written as x' & if value is '0' it is written as x

(6) $[x \wedge (y \vee (x \wedge \bar{y}))] \vee [(x \wedge \bar{y}) \vee (x \wedge \bar{z})]$ For $x=1, y=1, z=0$
$= [1 \wedge (1 \vee (1 \wedge 0))] \vee [(1 \wedge 0) \vee (\overline{1 \wedge 1})]$ $\bar{x}$ or $x' = 0$
$= [1 \wedge (1 \vee 0)] \vee [0 \vee \bar{1}] = [1 \wedge 1] \vee [0 \vee 0] = 1 \vee 0 = 1$ Ans $\bar{y}$ or $y' = 0$ & $\bar{z} = 1$

(6) $x + \overline{yz} = 0 + \overline{1 \cdot 1} = 0 + \bar{1} = 0 + 0 = 0$ (Here $x = 0$, $y = 1$ & $z = 1$)
Ans

8) Obtain the Sum of Products and Product-of-Sums Cannonical forms of the following:

(a) $x_1x_2' + x_3$ | (b) $[(x_1+x_2)(x_3x_4)']'$

(c) $x_1' + [x_2' + x_1 + (x_2x_3)'](x_2 + x_1'x_2)$

Sol: (a) $x_1x_2' + x_3$ (As per 2.4.9 on page 72) (as $x+x'=1$)

$x_1x_2'\cdot 1 + x_3\cdot 1 = x_1x_2'(x_3+x_3') + x_3(x_1+x_1')$

$= x_1x_2'x_3 + x_1x_2'x_3' + x_3x_1 + x_1'x_3$

$= x_1x_2'x_3 + x_1x_2'x_3' + x_1x_3(x_2+x_2') + x_1'x_3(x_2+x_2')$

$= x_1x_2'x_3 + x_1x_2'x_3' + x_1x_2x_3 + x_1x_2'x_3 + x_1'x_2x_3 + x_1'x_2'x_3$

$= x_1x_2'x_3 + x_1x_2'x_3' + x_1'x_2'x_3 + x_1'x_2x_3 + x_1x_2x_3$ ($\because P+P=P$)

which is required SOP Cannonical form.

For PoS form (as per 2.4.10 on Page 72 of Text book)

$x_1x_2' + x_3 = (x_1+x_3)(x_2'+x_3)$

$= (x_1+x_3+0)(x_2'+x_3+0)$

$= (x_1+x_3+x_2x_2')(x_2'+x_3+x_1x_1')$ (as $xx'=0$)

$= (x_1+x_2+x_3)(x_1+x_2'+x_3)(x_1+x_2'+x_3)(x_1'+x_2'+x_3)$

$= (x_1+x_2+x_3)(x_1+x_2'+x_3)(x_1'+x_2'+x_3)$ Ans ($\because S.S=S$)

which is required PoS Cannonical form

These forms can also be got by using Table method as shown on next page.

OR

8) (a) ordered table of the three variables for following function is

$x_1 x_2' + x_3$

x_1	x_2	x_3	x_2'	$x_1 x_2'$	$x_1 x_2' + x_3$	Min Terms	Max-term.
0	0	0	1	0	0	m_0	M_0
0	0	1	1	0	1	m_1	M_1
0	1	0	0	0	0	m_2	M_2
0	1	1	0	0	1	m_3	M_3
1	0	0	1	1	1	m_4	M_4
1	0	1	1	1	1	m_5	M_5
1	1	0	0	0	0	m_6	M_6
1	1	1	0	0	1	m_7	M_7

The minterms Corresponding to each ordered triple of 0's & 1 for which value of the function is 1 are m_1, m_3, m_4, m_5 & m_7

$m_1 = x_1' \wedge x_2' \wedge x_3$, $m_3 = x_1' \wedge x_2 \wedge x_3$, $m_4 = x_1 \wedge x_2' \wedge x_3'$

$m_5 = x_1 \wedge x_2' \wedge x_3$ & $m_7 = x_1 \wedge x_2 \wedge x_3$

$\therefore$ Sum of product form $= m_1 \vee m_3 \vee m_4 \vee m_5 \vee m_7$

$= (x_1' \wedge x_2' \wedge x_3) \vee (x_1' \wedge x_2 \wedge x_3) \vee (x_1 \wedge x_2' \wedge x_3') \vee (x_1 \wedge x_2' \wedge x_3) \vee (x_1 \wedge x_2 \wedge x_3)$ Ans

The Maxterms for which Value of the function is 0 are M_0, M_2 & M_6 where $M_0 = x_1 \vee x_2 \vee x_3$

$M_2 = x_1 \vee x_2' \vee x_3$, $M_6 = x_1' \vee x_2' \vee x_3$

$\therefore$ POS form of given function is $= M_0 \wedge M_2 \wedge M_6$

$= (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_2' \vee x_3) \wedge (x_1' \vee x_2' \vee x_3)$

Do the other parts Yourself (Try Them with both methods shown in part (a))

(9) (a) $f(x, y) = xy + x'y + x'y'$

Since it is two variable function therefore There will be $2^2 = 4$ cells in Karnough map. (K-map)

$x \backslash y$	0	1
0	$x'y'$	$x'y$
1	xy'	xy

$\therefore$ K-map for the given function is

$x \backslash y$	0	1
0	1	1
1	0	1

(b) $f(x, y) = xy' + x'y$

K- map of the given function is

$x \backslash y$	0	1
0	0	1
1	1	0

(7)

x_1	x_2	x_1'	$x_1 \wedge x_2$	$(x_1' \vee x_2)$	$x_1 \wedge (x_1' \vee x_2)$	$x_1 \vee (x_1 \wedge x_2)$
0	0	1	0	1	0	0
0	1	1	0	1	0	0
1	0	0	0	0	0	1
1	1	0	1	1	1	1